

IIT-JEE-Mathematics–Screening-2005

SCREENING

1. The area of the equilateral triangle, in which three coins of radius 1 cm are placed, as shown in the figure, is:

- (a) $6 + 4\sqrt{3}$
- (b) $4\sqrt{3} - 6$
- (c) $7 + 4\sqrt{3}$
- (d) $4\sqrt{3}$

2. The area bounded by the curves $y = (x - 1)^2$, $y = (x + 1)^2$ and $y = 1/4$ is:

- (a) $1/3$
- (b) $2/3$
- (c) $1/4$
- (d) $1/5$

3. The value of $\int_2^0 [x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)] dx$ is:

- (a) 0
- (b) 3
- (c) 4
- (d) 1

4. The tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at :

- (a) (6, 7)
- (b) (-6, 7)
- (c) (6, -7)
- (d) (-6, -7)

5. If $dy/dx = xy/(x^2 + y^2)$, $y(1) = 1$, then one of the values of x_0 satisfying $y(x_0) = e$ is given by

- (a) $e\sqrt{2}$
- (b) $e\sqrt{3}$
- (c) $e\sqrt{5}$
- (d) $e\sqrt{2}$

6. The locus of the centre of circle which touches $(y - 1)^2 + x^2 = 1$ externally also touches x axis is:

- (a) $x^2 = 4y$ \cap $(0, y), y < 0$
- (b) $x^2 = y$

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- (c) $y = 4x^2$
 (d) $y^2 = 4x \in (0, y), y \in \mathbb{R}$

7. If $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x$? $x \in [0, \pi/2]$ then $f(1/3)$ is:

- (a) 3
 (b) $\pi/3$
 (c) $1/3$
 (d) none of these

8.

$$\binom{30}{0} \binom{30}{10} - \binom{30}{1} \binom{30}{11} + \dots + \binom{30}{20} \binom{30}{30} =$$

- (a) ${}^{30}C_{11}$
 (b) ${}^{60}C_{10}$
 (c) ${}^{30}C_{10}$
 (d) ${}^{65}C_{55}$

9. A variable plane $x/a + y/b + z/c = 1$ at a unit distance from origin cuts the coordinate axes at A, B and C. Centroid (x, y, z) satisfies the equation $1/x^2 + 1/y^2 + 1/z^2 = K$. The value of K is :

- (a) 9
 (b) 3
 (c) $1/9$
 (d) $1/3$

10. Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and $D = b^2 - 4ac$. If $a + b$, $a^2 + b^2$ and $a^3 + b^3$ are in G.P., then :

- (a) $\Delta \neq 0$
 (b) $b\Delta \neq 0$
 (c) $c\Delta \neq 0$
 (d) $bc \neq 0$

11. Tangent at a point of the ellipse $x^2/a^2 + y^2/b^2 = 1$ is drawn which cuts the coordinate axes at A and B. The minimum area of the triangle OAB is (O being the origin) :

- (a) ab
 (b) $(a^3 + ab + b^3)/3$
 (c) $a^2 + b^2$
 (d) $((a^2 + b^2))/4$

12. A fair die is rolled. The probability that the first time 1 occurs at the even throw is :

- (a) $1/6$
 (b) $5/11$
 (c) $6/11$
 (d) $5/36$

13. If $x dy = y (dx + y dy)$, $y(1) = 1$ and $y(x) > 0$. Then $y(-3) =$:

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- (a) 3
- (b) 2
- (c) 1
- (d) 0

14. $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$ and

$g(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational.} \end{cases}$ Then $f - g$ is:

- (a) one-one and into
- (b) neither one-one nor onto
- (c) many one and onto
- (d) one-one and onto

15. A rectangle with sides $(2n - 1)$ and $(2m - 1)$ is divided into squares of unit length. The number of rectangle which can be formed with sides of odd length is :

- (a) $m^2 n^2$
- (b) $mn(m + 1)(n + 1)$
- (c) 4^{m+n-1}
- (d) none of these

16. The minimum value of $|a + b\omega + c\omega^2|$, where a, b and c are all not equal integers and $\omega (\neq 1)$ is a cube root of unity, is:

- (a) ?3
- (b) $1/3$
- (c) 1
- (d) 0

17. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2005} P$ is:

- (a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$
- (b) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$
- (c) $\begin{bmatrix} 1 & 0 \\ 2005 & 1 \end{bmatrix}$
- (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

18. The shaded region, where

$P ? (-1, 0)$, $Q ? (-1 + ?2, ?2)$

$R ? (-1 + ?2, - ?2)$, $S ? (1, 0)$ is represented by:

- (a) $|z + 1| > 2, |\arg(z + 1)| < \pi/4$

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- (b) $|z + 1| < 2, |\arg(z + 1)| < \pi/2$
- (c) $|z - 1| > 2, |\arg(z + 1)| > \pi/4$
- (d) $|z - 1| < 2, |\arg(z + 1)| > \pi/2$

19. The number of ordered pairs (α, β) , where $\alpha, \beta \in (-\Pi, \Pi)$ satisfying $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ is :

- (a) 0
- (b) 1
- (c) 2
- (d) 4

20. Let $f(x) = |x| - 1$, then points where $f(x)$ is not differentiable is/(are) :

- (a) $0, \pm 1$
- (b) ± 1
- (c) 0
- (d) 1

21. The second degree polynomial $f(x)$, satisfying $f(0) = 0, f(1) = 1, f'(x) > 0$ for all $x \in (0, 1)$:

- (a) $f(x) = \phi$
- (b) $f(x) = ax + (1 - a)x^2; ? a \in (0, \infty)$
- (c) $f(x) = ax + (1 - a)x^2; ? a \in (0, 2)$
- (d) no such polynomial

22. If f is a differentiable function satisfying $f(1/n) = 0$ for all $n \geq 1, n \in \mathbb{I}$, then :

- (a) $f(x) = 0, x \in (0, 1]$
- (b) $f'(0) = 0 = f(0)$
- (c) $f(0) = 0$ but $f'(0)$ not necessarily zero
- (d) $|f(x)| < 1, x \in (0, 1]$

23. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$,

$6A^{-1} = A^2 + cA + dI$, then (c, d) is:

- (a) $(-6, 11)$
- (b) $(-11, 6)$
- (c) $(11, 6)$
- (d) $(6, 11)$

24. In a ΔABC , among the following which one is true?

- (a) $(b + c) \cos A/2 = a \sin ((B+C)/2)$
- (b) $(b + c) \cos ((B+C)/2) = a \sin A/2$
- (c) $(b - c) \cos ((B-C)/2) = a \cos (A/2)$
- (d) $(b - c) \cos A/2 = a \cos ((B-C)/2)$

25. If $\vec{a}, \vec{b}, \vec{c}$ are three non zero, non coplanar vectors and $\vec{b}_1 = \vec{b} - \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \vec{a}$,
 $\vec{b}_2 = \vec{b} + \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \vec{a}$. And $\vec{c}_1 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} - \frac{\vec{c} \cdot \vec{b}_1}{|\vec{b}_1|^2} \vec{b}_1$, $\vec{c}_2 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} - \frac{\vec{c} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$,
 $\vec{c}_3 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} - \frac{\vec{c} \cdot \vec{b}}{|\vec{b}_2|^2} \vec{b}_2$, $\vec{c}_4 = \vec{a} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a}$.

Then which of the following is a set of mutually orthogonal vectors:

- (a) $(\vec{a}, \vec{b}_1, \vec{c}_1)$
 (b) $(\vec{a}, \vec{b}_1, \vec{c}_2)$
 (c) $(\vec{a}, \vec{b}_2, \vec{c}_3)$
 (d) $(\vec{a}, \vec{b}_2, \vec{c}_4)$
26. If $y = f(x)$ and $y \cos x + x \cos y = \Pi$, then the value of $f'(0)$ is :
 (a) Π
 (b) $-\Pi$
 (c) 0
 (d) 2Π
27. Let f be twice differentiable function satisfying $f(1) = 1$, $f(2) = 4$, $f(3) = 9$, then :
 (a) $f''(x) = 2, \forall x \in (\mathbb{R})$
 (b) $f''(x) = 5 = f''(x)$, for some $x \in (1, 3)$
 (c) There exists at least one $x \in (1, 3)$ such that $f''(x) = 2$
 (d) none of these
28. If X and Y are two non-empty sets where $f : X \rightarrow Y$ is function is defined such that
 $f(C) = \{f(x) : x \in C\}$ for $C \subseteq X$
 and $f^{-1}(D) = \{x : f(x) \in D\}$ for $D \subseteq Y$,
 for any $A \subseteq X$ and $B \subseteq Y$ then :
 (a) $f^{-1}(f(A)) = A$
 (b) $f^{-1}(f(A)) = A$ only if $f(X) = Y$
 (c) $f(f^{-1}(B)) = B$ only if $B \subseteq f(X)$
 (d) $f(f^{-1}(B)) = B$

IIT-JEE-Mathematics–Mains-2005

Mains

1. A person goes to office either by car, scooter, bus or train probability of which being $1/7, 3/7, 2/7$ and $1/7$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $2/9, 1/9, 4/9$ and $1/9$ respectively. Given that he reaches office in

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time, then what is the probability that he travelled by a car?

2. Find the range of values of t for which

$$2 \sin t = (1-2x+5x^2)/(3x^2-2x-1), t \in [-\pi/2, \pi/2].$$

3. Circles with radii 3, 4 and 5 touch each other externally if P is the point of intersection of tangents to these circles at their points of contact. Find the distance of P from the point of contact.

4. Find the equation of the plane containing the line $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $1/\sqrt{6}$ from the point $(2, 1 - 1)$.

5. If $|f(x_1) - f(x_2)| < (x_1 - x_2)^2$, for all $x_1, x_2 \in \mathbb{R}$. Find the equation of tangent to the curve $y = f(x)$ at the point $(1, 2)$.

6. If total number of runs scored in n matches is $((n+1)/4)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k th match are given by $k \cdot 2^{n+1-k}$, where $1 \leq k \leq n$. Find n .

7. The area of the triangle formed by the intersection of a line parallel to x -axis and passing through (h, k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of point P .

8. Evaluate

$$\int_0^{\pi} e^{|\cos x|} (2 \sin (1/2 \cos x) + 3 \cos (1/2 \cos x)) \sin x \, dx.$$

9. Incident ray is along the unit vector v and the reflected ray is along the unit vector w . The normal is along unit vector a outwards. Express vector w in terms of vector a and v .

10. Tangents are drawn from any point on the hyperbola $x^2/9 - y^2/4 = 1$ to the circle $x^2 + y^2 = 9$. Find the locus of mid-point of the chord of contact.

11. Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $x^2/25 + y^2/4 = 1$. Also find the length of the intercept of the tangent between the coordinate axes.

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12. If length of tangent at any point on the curve $y = f(x)$ intercepted between the point and the x-axis is of length 1. Find the equation of the curve.

13. Find the area bounded by the curves
 $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$.

14. If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + 3i$. Find the other vertices of square.

15. If $f(x - y) = f(x) \cdot g(y) - f(y) \cdot g(x)$ and $g(x - y) = g(x) \cdot g(y) + f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$. If right hand derivative at $x = 0$ exists for $f(x)$. Find derivative of $g(x)$ at $x = 0$.

16. If $p(x)$ be a polynomial of degree 3 satisfying $p(-1) = 10$, $p(1) = -6$ and $p(x)$ has maximum at $x = -1$ and $p'(x)$ has minima at $x = 1$. Find the distance between the local maximum and local minimum of the curve.

17. $f(x)$ is a differentiable function and $g(x)$ is a double differentiable function such that $|f(x)| < 1$ and $f'(x) = g(x)$. If $f^2(0) + g^2(0) = 0$. Prove that there exists some $c \in (-3, 3)$ such that $g(c) \cdot f(c) < 0$.

18. If

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 & + & 3a \\ 3b^2 & + & 3b \\ 3c^2 & + & 3c \end{bmatrix},$$

$f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y = f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by $f(x)$ and chord AB.