

## IIT-JEE-Mathematics-Paper2-2008

1. A particle P starts from the point  $z_0 = 1 + 2i$ , where  $i = \sqrt{-1}$ . It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point  $z_1$  the particle moves  $\sqrt{2}$  units in the direction of the unit vector  $i + j$  and then it moves through an angle  $\pi/2$  in anticlockwise direction on a circle with centre at origin, to reach a point  $z_2$ . The point  $z_2$  is given by

- (A)  $6 + 7i$
- (B)  $-7 + 6i$
- (C)  $7 + 6i$
- (D)  $-6 + 7i$

2. Let the function  $g : (-\infty, \infty) \rightarrow (-\pi/2, \pi/2)$  be given by  $g(u) = 2 \tan^{-1}(e^u) - \pi/2$ . Then,  $g$  is

- (A) even and is strictly increasing in  $(0, \infty)$
- (B) odd and is strictly decreasing in  $(-\infty, \infty)$
- (C) odd and is strictly increasing in  $(-\infty, \infty)$
- (D) neither even or odd, but is strictly increasing in  $(-\infty, \infty)$

3. Consider a branch of the hyperbola

$$x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$$

with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

- (A)  $1 - \sqrt{2}/3$
- (B)  $\sqrt{3}/2 - 1$
- (C)  $1 + \sqrt{2}/3$
- (D)  $\sqrt{3}/2 + 1$

GRAVITY CLASSES

4. The area of the region between the curves  $y = \frac{1 + \sin x}{\cos x}$  and  $y = \frac{1 - \sin x}{\cos x}$  bounded by the lines  $x = 0$  and  $x = \frac{\pi}{4}$  is

(A)  $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

(B)  $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

(C)  $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

(D)  $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

5. Consider three points  $P = (-\sin(\beta-\alpha), -\cos \beta)$ ,  $Q = (\cos(\beta-\alpha), \sin \beta)$  and  $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$ , where  $0 < \alpha, \beta, \theta < \frac{\pi}{4}$ . Then,

(A) P lies on the line segment RQ

(B) Q lies on the line segment PR

(C) R lies on the line segment QP

(D) P, Q, R are non-collinear

6. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

(A) 2, 4 or 8

(B) 3, 6 or 9

(C) 4 or 8

(D) 5 or 10

7. Let two non-collinear unit vectors  $a$  and  $b$  form an acute angle. A point P moves so that at any time  $t$  the position vector  $OP$  (where O is the origin) is given by  $a \cos t + b \sin t$ . When P is farthest from origin O, let M be the length of vector  $OP$  and  $u$  be the unit vector along vector  $OP$ . Then,

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- (A)  $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$  and  $M = (1 + \hat{a} \cdot \hat{b})^{\frac{3}{2}}$   
 (B)  $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$  and  $M = (1 + \hat{a} \cdot \hat{b})^{\frac{3}{2}}$   
 (C)  $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$  and  $M = (1 + 2\hat{a} \cdot \hat{b})^{\frac{3}{2}}$   
 (D)  $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$  and  $M = (1 + 2\hat{a} \cdot \hat{b})^{\frac{3}{2}}$

8. Let

$$I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx, J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx.$$

Then, for an arbitrary constant C, the value of J - I equals.

- (A)  $\frac{1}{2} \log \left( \frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + C$   
 (B)  $\frac{1}{2} \log \left( \frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + C$   
 (C)  $\frac{1}{2} \log \left( \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + C$   
 (D)  $\frac{1}{2} \log \left( \frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right) + C$

9. Let  $g(x) = \log f(x)$  where  $f(x)$  is a twice differentiable positive function on  $(0, \infty)$  such that  $f(x+1) = x f(x)$ . Then, for  $N = 1, 2, 3, \dots$ ,

$$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$$

- (A)  $-4 \left\{ 1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N-1)^2} \right\}$   
 (B)  $4 \left\{ 1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N-1)^2} \right\}$   
 (C)  $-4 \left\{ 1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N+1)^2} \right\}$   
 (D)  $4 \left\{ 1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N+1)^2} \right\}$

10. Suppose four distinct positive numbers  $a_1, a_2, a_3, a_4$  are in G.P. Let  $b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3$  and  $b_4 = b_3 + a_4$ .

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STATEMENT-1: The numbers  $b_1, b_2, b_3, b_4$  are neither in A.P. nor in G.P.

and

STATEMENT-2: The numbers  $b_1, b_2, b_3, b_4$  are in H.P.

- (A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.
- (B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.
- (C) Statement-1 is True, Statement-2 is False
- (D) Statement-1 is False, Statement-2 is True

11. Let  $a, b, c, p, q$  be real numbers. Suppose  $\alpha, \beta$  are the roots of the equation  $x^2 + 2px + q = 0$  and  $\alpha, 1/\beta$  are the roots of the equation  $ax^2 + 2bx + c = 0$ , where  $\beta^2$  is not belongs to  $\{-1, 0, 1\}$ .

STATEMENT-1:  $(p^2 - q)(b^2 - ac) \geq 0$

and

STATEMENT-2:  $b \neq pa$  or  $c \neq qa$

- (A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.
- (B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.
- (C) Statement-1 is True, Statement-2 is False
- (D) Statement-1 is False, Statement-2 is True

12. Consider

$$L_1 : 2x + 3y + p - 3 = 0$$

$$L_2 : 2x + 3y + p + 3 = 0$$

## GRAVITY CLASSES

where  $p$  is a real number, and  $C : x^2 + y^2 + 6x + 10y + 30 = 0$

STATEMENT-1: If line  $L_1$  is a chord of circle  $C$ , then line  $L_2$  is not always a diameter of circle  $C$ .

and

STATEMENT-2: If line  $L_1$  is a diameter of circle  $C$ , then line  $L_2$  is not a chord of circle  $C$ .

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

13. Let a solution  $y = y(x)$  of the differential equation

$$x^2(x^2-1) dy - y^2(y^2-1) dx = 0$$

satisfy  $y(2) = 2/3$ .

STATEMENT-1:  $y(x) = \sec(\sec^{-1} x - \pi/6)$

and

STATEMENT-2:  $y(x)$  is given by

$$1/y = (2/3)/x - (1 - 1/x^2)$$

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Paragraph

Consider the function  $f : (-2, 2) \rightarrow (-2, 2)$  defined by

$$f(x) = \frac{(x^2 - ax + 1)}{(x^2 + ax + 1)}, 0 < a < 2.$$

GRAVITY CLASSES

14. Which of the following is true?

- (A)  $(2 + a)^2 f'(1) + (2 - a)^2 f'(-1) = 0$
- (B)  $(2 - a)^2 f'(1) - (2 + a)^2 f'(-1) = 0$
- (C)  $f'(1)f'(-1) = (2 - a)^2$
- (D)  $f'(1)f'(-1) = (2 + a)^2$

15. Which of the following is true?

- (A)  $f(x)$  is decreasing on  $(-1, 1)$  and has a local minimum at  $x = 1$
- (B)  $f(x)$  is increasing on  $(-1, 1)$  and has a local maximum at  $x = 1$
- (C)  $f(x)$  is increasing on  $(-1, 1)$  but has neither a local maximum nor a local minimum at  $x = 1$ .
- (D)  $f(x)$  is decreasing on  $(-1, 1)$  but has neither a local maximum nor a local minimum at  $x = 1$ .

16. Let

$$g(x) = \int_0^x (f(t))/(1+t^2) dt.$$

Which of the following is true?

- (A)  $g'(x)$  is positive on  $(-?, 0)$  and negative on  $(0, ?)$
- (B)  $g'(x)$  is negative on  $(-?, 0)$  and positive on  $(0, ?)$
- (C)  $g'(x)$  changes sign on both  $(-?, 0)$  and  $(0, ?)$
- (D)  $g'(x)$  does not change sign on  $(-?, ?)$

Paragraph

Consider the lines

$$L1 : (x+1)/3=(y+2)/1=(z+1)/2$$

$$L2 : (x-2)/1=(y+2)/2=(z-3)/3$$

17. The unit vector perpendicular to both  $L1$  and  $L2$  is

- (A)  $(-i - 7j + 7k)/99$
- (B)  $(-i - 7j + 5k)/593$
- (C)  $(-i - 7j + 5k)/593$
- (D)  $(7i - 7j + k)/99$

**GRAVITY CLASSES**

**18.** The shortest distance between L1 and L2 is

- (A) 0
- (B)  $17/\sqrt{3}$
- (C)  $41/(5\sqrt{3})$
- (D)  $17/(5\sqrt{3})$

**19.** The distance of the point (1, 1, 1) from the plane passing through the point (-1, -2, -1) and whose normal is perpendicular to both the lines L1 and L2 is

- (A)  $2/\sqrt{75}$
- (B)  $7/\sqrt{75}$
- (C)  $13/\sqrt{75}$
- (D)  $23/\sqrt{75}$

**20.** Consider the lines given by

$$L_1 : x + 3y - 5 = 0$$

$$L_2 : 3x - ky - 1 = 0$$

$$L_3 : 5x + 2y - 12 = 0$$

Match the Statements/Expressions in Column I with the Statements/ Expressions in Column II.

Column I		Column II	
(A)	$L_1, L_2, L_3$ are concurrent if	(p)	$k = -9$
(B)	One of $L_1, L_2, L_3$ is parallel to at least one of the other two, if	(q)	$k = -6/5$
(C)	$L_1, L_2, L_3$ form a triangle if	(r)	$k = 5/6$
(D)	$L_1, L_2, L_3$ do not form a triangle if	(s)	$k = 5$

**21.** Match the Statements/Expressions in Column I and the Statements/ Expressions in Column II.

Column I		Column II	
(A)	The minimum value of $x^2+2x+4 / x+2$ is	(p)	0

## GRAVITY CLASSES

(B)	Let A and B be $3 \times 3$ matrices of real numbers, where A is symmetric, b is skew-symmetric, and $(A + B)(A - B) = (A - B)(A + B)$ . If $(AB)^t = (-1)^k AB$ , where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q)	1
(C)	Let $a = \log_3 \log_3 2$ . An integer k satisfying $1 < 2^{(-k+3^a)} < 2$ , must be less than	(r)	2
(D)	If $\sin q = \cos f$ , then the possible values of $1/\Pi(\theta \pm \emptyset - \Pi/2)$ are	(s)	3

22. Consider all possible permutations of the letters of the word ENDEANOEL.

Match the Statements/Expressions in Column I and the Statements/ Expressions in Column II.

Column I		Column II	
(A)	The number of permutations containing the word ENDEA is	(p)	$5!$
(B)	The number of permutations in which the letter E occurs in the first and the last positions is	(q)	$2 \times 5!$
(C)	The number of permutations in which one of the letters, D, L, N occurs in the last five positions is	(r)	$7 \times 5!$
(D)	The number of permutations in which the letters A, E, O occur only in odd positions is	(s)	$21 \times 5!$