

IIT-JEE-2008-Paper1

PAPER - I

SECTION - I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

1. If $0 < x < 1$ then $\sqrt{1+x^2} [\{x \cos(\cot^{-1} x) + \sin(\cot^{-1} x)\}^2 - 1]^{1/2}$

- (1) $x/\sqrt{1+x^2}$
- (2) x
- (3) $x\sqrt{1+x^2}$
- (4) $\sqrt{1+x^2}$

2. Consider the two curves

$$C_1 : y^2 = 4x$$

$$C_2 : x^2 + y^2 - 6x + 1 = 0$$

Then,

- (1) C_1 and C_2 touch each other only at one point
- (2) C_1 and C_2 touch each other exactly at two points
- (3) C_1 and C_2 intersect (but do not touch) at exactly two points
- (4) C_1 and C_2 neither intersect nor touch each other

3. The edges of a parallelepiped are of unit length and are parallel to non-coplanar unit vectors a, b, c such that $a \cdot b = b \cdot c = c \cdot a = 1/2$

Then, the volume of the parallelepiped is

- (1) $1/\sqrt{2}$
- (2) $1/2\sqrt{2}$
- (3) $\sqrt{3}/2$
- (4) $1/\sqrt{3}$

4. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents

- (1) four straight lines, when $c = 0$ and a, b are of the same sign
- (2) two straight lines and a circle, when $a = b$, and c is of sign opposite to that of a
- (3) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a
- (4) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to

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that of a.

5. The total number of local maxima and minima of the function

$$f(x) = \begin{cases} (2+x)^3, & -3 < x \leq -1 \\ x^{2/3}, & -1 < x < 2 \end{cases} \text{ is}$$

- (1) 0
- (2) 1
- (3) 2
- (4) 3

6.

$$\text{Let } g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}; 0 < x < 2,$$

m and n are integers $m \neq 0, n > 0$ and let p be the left hand derivative of

$$|x-1| \text{ at } x=1. \text{ If } \lim_{x \rightarrow 1^+} g(x) = p, \text{ then}$$

- (1) $n = 1, m = 1$
- (2) $n = 1, m = -1$
- (3) $n = 2, m = 2$
- (4) $n > 2, m = n$

SECTION II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONE OR MORE is/are correct.

7. Let $P(x_1, y_1)$ and $Q(x_2, y_2), y_1 < 0, y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

- (1) $x^2 + 2y = 3 + \sqrt{3}$
- (2) $x^2 - 2y = 3 + \sqrt{3}$
- (3) $x^2 + 2y = 3 - \sqrt{3}$
- (4) $x^2 - 2y = 3 - \sqrt{3}$

8. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T . If S is not the centre of the circumcircle, then

- (1) $1/PS + 1/ST < 2/(QS * SR)$
- (2) $1/PS + 1/ST > 2/(QS * SR)$
- (3) $1/PS + 1/ST < 4/QR$

9. Let $f(x)$ be a non-constant twice differentiable function defined

$$(-\infty, \infty) \text{ such that } f(x) = f(1-x) \text{ and } f\left(\frac{1}{4}\right) = 0$$

Then,

(1) $f'(x)$ vanishes at least twice on $[0, 1]$

$$(2) f\left(\frac{1}{2}\right) = 0$$

$$(3) \int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right) \sin x \, dx = 0$$

$$(4) \int_0^{1/2} f(t) e^{\sin t} \, dt = \int_{1/2}^1 f(1-t) e^{\sin t} \, dt$$

10.

$$\text{Let } S_n = \sum_{k=1}^n \frac{n}{n^2 + kn + k^2} \text{ and } T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2} \text{ for } n = 1, 2, 3, \dots \text{ then,}$$

$$(1) S_n < \pi/3? 3$$

$$(2) S_n > \pi/3? 3$$

$$(3) T_n < \pi/3? 3$$

$$(4) T_n > \pi/3? 3$$

SECTION - III

Assertion - Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

11. Consider the system of equations

$$x - 2y + 3z = -1$$

$$-x + y - 2z = k$$

$$x - 3y + 4z = 1$$

STATEMENT-1: The system of equations has no solutions for $k \neq 3$ and

STATEMENT-2:

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$$\text{The determinant } \begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0, \text{ for } k \neq 3$$

- (1) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
- (2) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
- (3) STATEMENT-1 is True, STATEMENT-2 is False
- (4) STATEMENT-1 is False, STATEMENT-2 is True

12. Consider the system of equations $ax + by = 0$, $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$

STATEMENT-1: The probability that the system of equations has a unique solution is $\frac{3}{8}$
and

STATEMENT-2: The probability that the system of equations has a solution is 1.

- (1) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
- (2) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
- (3) STATEMENT-1 is True, STATEMENT-2 is False
- (4) STATEMENT-1 is False, STATEMENT-2 is True

13. Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is continuous $g(0) \neq 0$, $g''(0) = 0$

STATEMENT-1: $g''(0) \neq 0$, and $f(x) = g(x) \sin x$
 $\lim_{x \rightarrow 0} [g(x) \cot x - g(x) \operatorname{cosec} x] = f'(0)$.
and

STATEMENT-2: $f'(0) = g(0)$

- (1) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
- (2) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
- (3) STATEMENT-1 is True, STATEMENT-2 is False
- (4) STATEMENT-1 is False, STATEMENT-2 is True

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$$P_3 : x - 3y + 3z = 2$$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3 , P_3 and P_1 , and P_1 and P_2 , respectively

STATEMENT-1: At least two of the lines L_1, L_2 and L_3 are non-parallel and

STATEMENT-2: The three planes do not have a common point

- (1) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(2) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(3) STATEMENT-1 is True, STATEMENT-2 is False
(4) STATEMENT-1 is False, STATEMENT-2 is True

SECTION - IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 15 to 17

Let A, B, C be three sets of complex numbers as defined below

$$A = \{ z : \text{Im}z \geq 1 \}$$

$$B = \{ z : |z-2-i| = 3 \}$$

$$C = \{ z : \text{Re}((1-i)z) = 2 \}$$

15. The number of element in the set $A \cap B \cap C$ is

- (1) 0
(2) 1
(3) 2
(4) ?

16. Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

- (1) 25 and 29
(2) 30 and 34
(3) 35 and 39
(4) 40 and 44

- (1) -6 and 3
- (2) -3 and 6
- (3) -6 and 6
- (4) -3 and 9

Paragraph for Questions Nos. 18 to 20

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $(\sqrt{3}/2, 3/2)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

18. The equation of circle C is

- (1) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$
- (2) $(x - \sqrt{3})^2 + (y - 1/2)^2 = 1$
- (3) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$
- (4) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

19. Points E and F are given by

- (1) $(\sqrt{3}/2, 3/2)$ $(\sqrt{3}, 0)$
- (2) $(\sqrt{3}/2, 1/2)$ $(\sqrt{3}, 0)$
- (3) $(\sqrt{3}/2, 3/2)$ $(\sqrt{3}/2, 1/2)$
- (4) $(3/2, \sqrt{3}/2)$ $(\sqrt{3}/2, 1/2)$

20. Equations of the sides QR, RP are

- (1) $y = (2/\sqrt{3})x + 1, y = -(2/\sqrt{3})x - 1$
- (2) $y = (1/\sqrt{3})x, y = 0$
- (3) $y = (\sqrt{3}/2)x + 1, y = -(\sqrt{3}/2)x - 1$
- (4) $y = (\sqrt{3})x, y = 0$

Paragraph for Questions Nos. 21 to 23

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\sqrt{3}, \sqrt{3}) \cup (\sqrt{3}, \infty)$ the equation implicitly defines a unique real valued differentiable function $y = f(x)$.

If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

21. If $f(\sqrt{3}) = \sqrt{3}$, then $f'(\sqrt{3}) =$

- (1) $4\sqrt{3} / 7^3 \cdot 3^2$
- (2) $-4\sqrt{3} / 7^3 \cdot 3^2$
- (3) $4\sqrt{3} / 7^3 \cdot 3$
- (4) $-4\sqrt{3} / 7^3 \cdot 3$

22. The area of the region bounded by the curve $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$, where $-2 < a < b < 2$, is

$$(1) \int_a^b \frac{x}{3((f(x))^2 - 1)} dx + bf(b) - af(a)$$

$$(2) -\int_a^b \frac{x}{3((f(x))^2 - 1)} dx + bf(b) - af(a)$$

$$(3) \int_a^b \frac{x}{3((f(x))^2 - 1)} dx - bf(b) + af(a)$$

$$(4) -\int_a^b \frac{x}{3((f(x))^2 - 1)} dx - bf(b) + af(a)$$

23. $\int_1^{-1} g'(x) dx =$

$$(1) 2g(-1)$$

$$(2) 0$$

$$(3) -2g(1)$$

$$(4) 2g(1)$$