Practice the following Assignment

1. Find the minimum and maximum values of the following:

(a) $3\sin^2\theta + 5\cos^2\theta + 3\cos\theta\sin\theta$

(b) $4\sin^6\theta + 2\sin^4\theta + 3\sin^2\theta + 3\cos^2\theta + 2\cos^4\theta + 4\cos^6\theta$

(c) $a\sin^2\theta + b\cos^2\theta + c$ (d) $\sqrt{\tan^2\theta + 2\sin^2\theta} + \sqrt{5\cos^2\theta - \sin^2\theta}$

(e) $\sin^4\theta + \cos^2\theta$ (f) $\sin^2\theta + \cos^4\theta$ (g) $\sin^8\theta + \cos^8\theta$

(h) $2^{\sin x} + 2^{\cos x}$ (i) $9\tan^2\theta + 4\cot^2\theta$

(j) $\sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

2. Find the value of:

(a) $\sin 10^0 - \sin 15^0 + \sin 20^0 - \sin 25^0 ----\infty =$

(b) $\cos 10^0 - \cos 15^0 + \cos 20^0 - \cos 25^0 ----\infty =$

(c) $\tan 2^0 + 2\tan 4^0 + 4\tan 8^0 + 8\tan 16^0 + ---128\tan 256^0 =$

(d) $\sec^2 1^0 + 4\sec^2 2^0 + 16\sec^2 4^0 + ------ + 65536\sec^2 256^0 =$

(e) $(\tan 9^0 \tan 63^0 \tan 81^0 \tan 87^0 - 1)\sin 3^0 \cos 63^0 =$

(g) $\sqrt{1-\cos\alpha} + \sqrt{1-\cos 2\alpha} + \sqrt{1-\cos 3\alpha} + ---- + \sqrt{1-\cos n\alpha} =$

(h) $\sin\frac{\pi}{7}.\sin\frac{2\pi}{7}.\sin\frac{3\pi}{7} =$

(i) $\left(1+\cos\frac{\pi}{9}\right)\left(1+\cos\frac{3\pi}{9}\right)\left(1+\cos\frac{5\pi}{9}\right)\left(1+\cos\frac{7\pi}{9}\right)$

(j) $\sin 55^0 - \sin 19^0 + \sin 53^0 - \sin 17^0 = \cos 1^0$ [Prove it]

(k) $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$ (l) $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$

(m) $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}......(7 \text{ factors}) = \frac{1}{2^7}$ (n) $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}......(7 \text{ factors}) = \frac{1}{2^6}$

(o) $\operatorname{cosec}\theta + \operatorname{cosec}2\theta + \operatorname{cosec}4\theta + ... + \operatorname{cosec}8\theta = \cot\frac{\theta}{2} - \cot 8\theta$ [this is an important result]

(p) $\sin\alpha - \sin 2\alpha + \sin 3\alpha$........ n terms (q) $\sum_{r=1}^{10}\cos^3\frac{r\pi}{3}$ (r) $\sum_{r=1}^{9}\sin^2\frac{r\pi}{18}$

(s) $\sqrt{1+\sin 2\alpha} + \sqrt{1+\sin 4\alpha} + \sqrt{1+\sin 6\alpha} +$......... 20 terms

(t) Prove that: $\sin^2 12^0 + \sin^2 21^0 + \sin^2 39^0 + \sin^2 48^0 = 1 + \sin^2 9^0 + \sin^2 18^0$

3. Prove that:

(a) $(1+\sec 2\theta)(1+\sec 2^2\theta)(1+\sec 2^3\theta)....(1+\sec 2^n\theta) = \tan 2^n\theta\cot\theta$

(b) If $xy + yz + zx = 1$, show that

$$\frac{x}{1-x^2} + \frac{y}{1-y^2} + \frac{z}{1-z^2} = \frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$$

Trigonometry

Some important results

$* \ \sin^2 A = \dfrac{1-\cos 2A}{2}$ $\quad * \ \cos^2 A = \dfrac{1+\cos 2A}{2}$ $\quad * \ \tan^2 A = \dfrac{1-\cos 2A}{1+\cos 2A}$

$* \ \sin^3 A = \dfrac{3\sin A - \sin 3A}{4}$ $\quad * \ \cos^3 A = \dfrac{3\cos A - \cos 3A}{4}$

$\rightarrow \sin^4 A + \cos^4 A = 1 - 2\sin^2 A\cos^2 A$

$\rightarrow \sin^6 A + \cos^6 A = 1 - 3\sin^2 A\cos^2 A$

$* \ \dfrac{\cos^4\theta}{a} + \dfrac{\sin^4\theta}{b} = \dfrac{1}{a+b} \quad \Rightarrow \quad \cos\theta = \dfrac{\sqrt{a}}{\sqrt{a+b}};\ \sin\theta = \dfrac{\sqrt{b}}{\sqrt{a+b}};\ \tan\theta = \dfrac{\sqrt{b}}{\sqrt{a}}$

$* \ \dfrac{\cos^8\theta}{a^3} + \dfrac{\sin^8\theta}{b^3} = \dfrac{1}{(a+b)^3} \quad \Rightarrow \quad \cos\theta = \dfrac{\sqrt{a}}{\sqrt{a+b}};\ \sin\theta = \dfrac{\sqrt{b}}{\sqrt{a+b}};\ \tan\theta = \dfrac{\sqrt{b}}{\sqrt{a}}$

These results are generally important a short tricks :-

Example 1 : $\dfrac{\cos^4\theta}{5} + \dfrac{\sin^4\theta}{3} = \dfrac{1}{8}$. Find $\cos 2\theta = ?$

Solution : $\cos^2\theta - \sin^2\theta = \dfrac{5}{8} - \dfrac{3}{8} = \dfrac{2}{8} = \dfrac{1}{4}$

Example 2 : $8\cos^8\theta + \sin^8\theta = \dfrac{8}{27}$; Find $\sin 3\theta = ?$

Solution : $\cos^8\theta + \dfrac{\sin^8\theta}{2^3} = \dfrac{1}{(1+2)^3} \quad \Rightarrow \cos^2\theta = \dfrac{1}{3};\ \sin^2\theta = \dfrac{2}{3}$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta = \sin\theta(3 - 4\sin^2\theta)$

$= \sqrt{\dfrac{2}{3}}\left(3 - 4\times\dfrac{2}{3}\right) = \dfrac{\sqrt{2}}{3\sqrt{3}}$ Ans.

Example 3 : Find the value of $\cot 7\dfrac{1}{2}^0$.

Solution : $\cot 7\dfrac{1}{2}^0 = \cot\left(\dfrac{15^0}{2}\right) = \cot\left(\dfrac{45^0 - 30^0}{2}\right)$

$\cot 15^0 = \dfrac{\cot 45^0 \cot 30^0 + 1}{\cot 30^0 - \cot 45^0} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \quad \Rightarrow \cos 15^0 = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$\cot 7\dfrac{1}{2}^0 = \sqrt{\dfrac{1+\cos 15^0}{1-\cos 15^0}} = \sqrt{\dfrac{2\sqrt{2}+\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}-1}}$ Ans.

Note : Whenever we have to calculate any trigonometric function $\left(\theta/2\right)$.

Use : $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$; $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$; $\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$;

Also we should remember following values.

$0^0, 15^0, 18^0, 30^0, 36^0, 37^0, 45^0, 53^0, 54^0, 60^0, 72^0, 75^0, 90^0$

$\cos 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}}$	$\sin 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$	$\tan 15^0 = \frac{\sqrt{3}-1}{\sqrt{3}+1}$