QUADRATIC EQUATION ASSIGNMENT

1. If $-1 \le x < 0$, then solution of the equation $|x+1| - |x| + 3|x-1||x-2| = x+2$ is

(a) 1, 5/3 (b) 5/3 (c) 1/3 (d) None of these

2. The real roots of $|x|^3 - 3x^2 + 3|x| - 2 = 0$ are

(a) 0, 2 (b) ± 1 (c) ± 2 (d) 1, 2

3. The number of real solutions of the equation $2^{x/2} + (\sqrt{2}+1)^x = (5+2\sqrt{2})^{x/2}$ is

(a) One (b) Two (c) Four (d) Infinite

4. The number of negative integral solutions of $x^2.2^{x+1} + 2^{|x-3|+2} = x^2.2^{|x-3|+4} + 2^{x-1}$ is

(a) 0 (b) 1 (c) 2 (d) 4

5. The equation $e^x - x - 1 = 0$ has

(a) Only one real root $x = 0$ (b) At least two real roots (c) Exactly two real roots (d) Infinitely many real roots

6. The number of real roots of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ are

(a) 1 (b) 2 (c) Infinite (d) None of these

7. If *a*, *b*, *c* are positive real numbers, then the number of real roots of the equation $ax^2 + b|x| + c = 0$ is

(a) 2 (b) 4 (c) 0 (d) None of these

8. The number of real solutions of equation $\log_{10}[98 + \sqrt{[x^3 - x^2 - 12x + 36]}] = 2$ are

(a) 4 (b) 1 (c) 2 (d) 3

9. The equation $x^{(3/4)(\log_2 x)^2 + (\log_2 x) - 5/4} = \sqrt{2}$ has

(a) At least one real solution (b) Exactly three real solutions

(c) Exactly one irrational solution (d) All the above

10. The number of solutions of $|[x] - 2x| = 4$, where [*x*] is the greatest integer is $\le x$, is

(a) 2 (b) 4 (c) 1 (d) Infinite

11. Let *f*(*x*) be a function defined by $f(x) = x - [x]$, $0 \ne x \in R$, where [*x*] is the greatest integer less than or equal to *x*. then the number of solutions of $f(x) + f\left(\frac{1}{x}\right) = 1$

(a) 0 (b) Infinite (c) 1 (d) 2

12. If *m* be the number of integral solutions of equation $2x^2 - 3xy - 9y^2 - 11 = 0$ and *n* be the number of real solutions of equation $x^3 - [x] - 3 = 0$, then *m* =

(a) *n* (b) 2*n* (c) *n*/2 (d) 3*n*

13. The set of values of *c* for which $x^3 - 6x^2 + 9x - c$ is of the form $(x-\alpha)^2(x-\beta)$ (α, β real) is given by

(a) {0} (b) {4} (c) {0, 4} (d) Null set

14. If $0 < a_r < 1$ for *r* = 1, 2, 3, ..., *k* and *m* be the number of real solutions of equation $\sum_{r=1}^{k} (a_r)^x = 1$ and *n* be the number of real solution of equation $\sum_{r=1}^{k} (x - a_r)^{101} = 0$, then

(a) $m = n$ (b) $m \le n$ (c) $m \ge n$ (d) $m > n$

15. Let $P_n(x) = 1 + 2x + 3x^2 + ... + (n+1)x^n$ be a polynomial such that *n* is even. Then the number of real roots of $P_n(x) = 0$ is

(a) 0 (b) *n* (c) 1 (d) None of these

16. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0$ for all *x* is

(a) Zero (b) One (c) Three (d) Infinite

17. The solutions of the equation $2x - 2[x] = 1$, where [*x*] = the greatest integer less than or equal to *x*, are

(a) $x = n + \frac{1}{2},\ n \in N$ (b) $x = n - \frac{1}{2},\ n \in N$ (c) $x = n + \frac{1}{2},\ n \in Z$ (d) $n < x < n+1,\ n \in Z$

18. The number of real solutions of $1 + |e^x - 1| = e^x(e^x - 2)$ is

(a) 0 (b) 1 (c) 2 (d) 4

19. The equation $2\sin^2\frac{x}{2} \cdot \cos^2 x = x + \frac{1}{x}, 0 < x \le \frac{\pi}{2}$ has

(a) One real solution (b) No real solution

(c) Infinitely many real solutions (d) None of these

20. The equation $(a+2)x^2 + (a-3)x = 2a - 1, a \ne -2$ has roots rational for

(a) All rational values of *a* except $a = -2$ (b) All real values of *a* except $a = -2$

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(c) Rational values of $a > \frac{1}{2}$ (d) None of these

21. The quadratic equation $x^2 - 2x - \lambda = 0, \lambda \neq 0$

(a) Cannot have a real root if $\lambda < 1$

(b) Can have a rational root if λ is a perfect square

(c) Cannot have an integral root if $n^2 - 1 < \lambda < n^2 + 2n$ where $n = 0, 1, 2, 3,....$

(d) None of these

22. If the roots of the equation $x^2 + px + q = 0$ are α and β and roots of the equation $x^2 - xr + s = 0$ are α^4, β^4, then the roots of the equation $x^2 - 4qx + 2q^2 - r = 0$ will be

(a) Both negative (b) Both positive

(c) Both real (d) One negative and one positive

23. If equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, $a, b, c > 0$, $n \in N$, then

(a) $a^n + c^n \geq 2b^n$ (b) $a^n + c^n > 2b^n$ (c) $a^n + c^n \leq 2b^n$ (d) $a^n + c^n < 2b^n$

24. If $\dfrac{\sum_{r=0}^{k-1} x^{2r}}{\sum_{r=0}^{k-1} x^r}$ is a polynomial in x for two values of p and q of k, then roots of equation $x^2 + px + q = 0$ cannot be

(a) Real (b) Imaginary (c) Rational (d) Irrational

25. If for $x > 0$, $f(x) = (a - x^n)^{1/n}$, $g(x) = x^2 + px + q, p, q \in R$ and equation $g(x) - x = 0$ has imaginary roots, then number of real roots of equation $g(g(x)) - f(f(x)) = 0$ is

(a) 0 (b) 2 (c) 4 (d) None of these

26. Let $p, q \in \{1, 2, 3, 4\}$. The number of equations of the form $px^2 + qx + 1 = 0$ having real and unequal roots is

(a) 15 (b) 9 (c) 7 (d) 8

27. If α_1, α_2 and β_1, β_2 are the roots of the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively and system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-zero solution. Then

(a) $a^2qc = p^2br$ (b) $p^2br = q^2ac$ (c) $c^2ar = r^2pb$ (d) None of these

28. If a, b, c, d are four consecutive terms of an increasing AP then the roots of the equation $(x-a)(x-c) + 2(x-b)(x-d) = 0$ are

(a) Real and distinct (b) Nonreal complex (c) Real and equal (d) Integers

29. If a, b, c are three distinct positive real numbers then the number of real roots of $ax^2 + 2b|x| - c = 0$ is

(a) 4 (b) 2 (c) 0 (d) None of these

30. If $a \in R$, $b \in R$ then the equation $x^2 - abx - a^2 = 0$ has

(a) One positive root and one negative root (b) Both roots positive

(c) Both roots negative (d) Non-real roots

31. The number of integral values of a for which $x^2 - (a-1)x + 3 = 0$ has both roots positive and $x^2 + 3x + 6 - a = 0$ has both roots negative is

(a) 0 (b) 1 (c) 2 (d) Infinite

32. The quadratic equations $x^2 + (a^2 - 2)x - 2a^2 = 0$ and $x^2 - 3x + 2 = 0$ have

(a) No common root for all $a \in R$ (b) Exactly one common root for all $a \in R$

(c) Two common roots for some $a \in R$ (d) None of these

33. If α, β are the roots of $x^2 + px + 1 = 0$ and γ, δ are the roots of $x^2 + qx + 1 = 0$, then $q^2 - p^2 =$

(a) $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$ (b) $(\alpha + \gamma)(\beta + \gamma)(\alpha - \delta)(\beta + \delta)$

(c) $(\alpha + \gamma)(\beta + \gamma)(\alpha + \delta)(\beta + \delta)$ (d) None of these

34. If α, β be the roots of $x^2 - px + q = 0$ and α', β' be the roots of $x^2 - p'x + q' = 0$, then the value of $(\alpha - \alpha')^2 + (\beta - \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2$ is

(a) $2\{p^2 - 2q + p'^2 - 2q' - pp'\}$ (b) $2\{p^2 - 2q + p'^2 - 2q' - qq'\}$

(c) $2\{p^2 - 2q - p'^2 - 2q' - pp'\}$ (d) $2\{p^2 - 2q - p'^2 - 2q' - qq'\}$

35. If α and β are the roots of the equation $x^2 - ax + b = 0$ and $A_n = \alpha^n + \beta^n$, then which of the following is true

(a) $A_{n+1} = aA_n + bA_{n-1}$ (b) $A_{n+1} = bA_n + aA_{n-1}$ (c) $A_{n+1} = aA_n - bA_{n-1}$ (d) $A_{n+1} = bA_n - aA_{n-1}$

36. If roots of an equation $x^n - 1 = 0$ are $1, a_1, a_2....., a_{n-1}$, then the value of $(1 - a_1)(1 - a_2)(1 - a_3).....(1 - a_{n-1})$ will be

(a) n (b) n^2 (c) n^n (d) 0

37. If α and β are the roots of $6x^2-6x+1=0$, then the value of $\frac{1}{2}[a+b\alpha+c\alpha^2+d\alpha^3]+\frac{1}{2}[a+b\beta+c\beta^2+d\beta^3]$ is

(a) $\frac{1}{4}(a+b+c+d)$ (b) $\frac{a}{1}+\frac{b}{2}+\frac{c}{3}+\frac{d}{4}$ (c) $\frac{a}{1}-\frac{b}{2}+\frac{c}{3}-\frac{d}{4}$ (d) None of these

38. If α_1, α_2 are the roots of equation $x^2-px+1=0$ and β_1, β_2 be those of equation $x^2-qx+1=0$ and vector $\alpha_1\hat{i}+\beta_1\hat{j}$ is parallel to $\alpha_2\hat{i}+\beta_2\hat{j}$, then

(a) $p=\pm q$ (b) $p=\pm 2q$ (c) $p=2q$ (d) None of these

39. If the sum of the roots of the equation $qx^2+2x+3q=0$ is equal to their product, then the value of q is equal to

(a) $-\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) –6

40. If $x=(\beta-\gamma)(\alpha-\delta),\ y=(\gamma-\alpha)(\beta-\delta),\ z=(\alpha-\beta)(\gamma-\delta)$, then the value of $x^3+y^3+z^3-3xyz$ is

(a) 0 (b) $\alpha^6+\beta^6+\gamma^6+\delta^6$ (c) $\alpha^6\beta^6\gamma^6\delta^6$ (d) None of these

41. If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, then $(1-\alpha^2)(1-\beta^2)(1-\gamma^2)$ is equal to

(a) $(1+q)^2-(p+r)^2$ (b) $(1+q)^2+(p+r)^2$ (c) $(1-q)^2+(p-r)^2$ (d) None of these

42. If α, β, γ are the roots of the equation $x^3+ax+b=0$, then $\frac{\alpha^3+\beta^3+\gamma^3}{\alpha^2+\beta^2+\gamma^2}=$

(a) $\frac{3b}{2a}$ (b) $\frac{-3b}{2a}$ (c) $3b$ (d) $2a$

43. If α, β are the roots of $6x^2-2x+1=0$ and $s_x=\alpha^n+\beta^n$, then $\lim_{n\to\infty}\sum_{r=1}^{n} S_r$ is

(a) $\frac{5}{17}$ (b) 0 (c) $\frac{3}{37}$ (d) None of these

44. Let α, β be the roots of the equation $ax^2+bx+c=0$ and let $\alpha^n+\beta^n=S_n$ for $n\ge 1$. Then the value of the determinant

$$\begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix} \text{ is}$$

(a) $\frac{b^2-4ac}{a^4}$ (b) $\frac{(a+b+c)(b^2+4ac)}{a^4}$ (c) $\frac{(a+b+c)(b^2-4ac)}{a^4}$ (d) $\frac{(a+b+c)^2(b^2-4ac)}{a^4}$

45. If α, β are roots of the equation $2x^2+6x+b=0\,(b<0)$, then $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is less then

(a) 2 (b) –2 (c) 18 (d) None of these

46. If α, β are roots of the equation $ax^2+3x+2=0\ (a<0)$, then $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ is greater then

(a) 0 (b) 1 (c) 2 (d) None of these

47. If $\alpha, \beta, \gamma, \sigma$ are the roots of the equation $x^4+4x^3-6x^2+7x-9=0$, then the value of $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)(1+\sigma^2)$ is

(a) 5 (b) 9 (c) 11 (d) 13

48. If α and β are the roots of the equation $x^2-p(x+1)-q=0$, then the value of $\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+q}+\frac{\beta^2+2\beta+1}{\beta^2+2\beta+q}$ is

(a) 2 (b) 3 (c) 0 (d) 1

49. If the roots of $ax^2-bx-c=0$ change by the same quantity then the expression in *a, b, c* that does not change is

(a) $\frac{b^2-4ac}{a^2}$ (b) $\frac{b-4c}{a}$ (c) $\frac{b^2+4ac}{a^2}$ (d) None of these

50. The number of positive integral solutions of $\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6}\le 0$ is

(a) 4 (b) 3 (c) 2 (d) 1