ELLIPSE ASSIGNMENT

1. An ellipse is described by using an endless string which is passed over two pins. If the axes are 6 *cm* and 4 *cm*, the necessary length of the string and the distance between the pins respectively in *cm,* are

(a) $6, 2\sqrt{5}$ (b) $6, \sqrt{5}$ (c) $4, 2\sqrt{5}$ (d) None of these

2. A man running round a race-course notes that the sum of the distances of two flag-posts from him is always 10 *meters* and the distance between the flag-posts is 8 *meters.* The area of the path he encloses in square metres is

(a) 15π (b) 12π (c) 18π (d) 8π

3. The equation $\frac{x^2}{1-r}-\frac{y^2}{1+r}=1$, $r>1$ represents

(a) An ellipse (b) A hyperbola (c) A circle (d) An imaginary ellipse

4. The radius of the circle having its centre at (0,3) and passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, is

(a) 3 (b) 3.5 (c) 4 (d) $\sqrt{12}$

5. The centre of an ellipse is *C* and *PN* is any ordinate and A, A' are the end points of major axis, then the value of $\frac{PN^2}{AN.A'N}$ is

(a) $\frac{b^2}{a^2}$ (b) $\frac{a^2}{b^2}$ (c) a^2+b^2 (d) 1

6. Let *P* be a variable point on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ with foci at *S* and S'. If *A* be the area of triangle PSS', then the maximum value of *A* is

(a) 24 sq. units (b) 12 sq. units (c) 36 sq. units (d) None of these

7. The eccentricity of the ellipse which meets the straight line $\frac{x}{7}+\frac{y}{2}=1$ on the axis of *x* and the straight line $\frac{x}{3}-\frac{y}{5}=1$ on the axis of *y* and whose axes lie along the axes of coordinates, is

(a) $\frac{3\sqrt{2}}{7}$ (b) $\frac{2\sqrt{6}}{7}$ (c) $\frac{\sqrt{3}}{7}$ (d) None of these

8. If the focal distance of an end of the minor axis of an ellipse (referred to its axes as the axes of *x* and *y* respectively) is *k* and the distance between its foci is 2*h*, then its equation is

(a) $\frac{x^2}{k^2}+\frac{y^2}{h^2}=1$ (b) $\frac{x^2}{k^2}+\frac{y^2}{k^2-h^2}=1$ (c) $\frac{x^2}{k^2}+\frac{y^2}{h^2-k^2}=1$ (d) $\frac{x^2}{k^2}+\frac{y^2}{k^2+h^2}=1$

9. If (5, 12) and (24, 7) are the foci of a conic passing through the origin, then the eccentricity of conic is

(a) $\frac{\sqrt{386}}{38}$ (b) $\frac{\sqrt{386}}{12}$ (c) $\frac{\sqrt{386}}{13}$ (d) $\frac{\sqrt{386}}{25}$

10. The maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the vertex at one end of the major axis is

(a) $\sqrt{3}ab$ (b) $\frac{3\sqrt{3}}{4}ab$ (c) $\frac{5\sqrt{3}}{4}ab$ (d) None of these

11. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having its centre (0, 3) is

(a) 4 (b) 3 (c) $\sqrt{12}$ (d) $\frac{7}{2}$

12. The locus of extremities of the latus rectum of the family of ellipse $b^2x^2+y^2=a^2b^2$ is

(a) $x^2-ay=a^2$ (b) $x^2-ay=b^2$ (c) $x^2+ay=a^2$ (d) $x^2+ay=b^2$

13. The parametric representation of a point on the ellipse whose foci are (–1, 0) and (7, 0) and eccentricity 1/2 is

(a) $(3+8\cos\theta, 4\sqrt{3}\sin\theta)$ (b) $(8\cos\theta, 4\sqrt{3}\sin\theta)$ (c) $(3+4\sqrt{3}\cos\theta, 8\sin\theta)$ (d) None of these

14. If $P(\theta)$ and $Q\left(\frac{\pi}{2}+\theta\right)$ are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then locus of the mid-point of *PQ* is

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$ (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=4$ (c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ (d) None of these

15. The locus of mid points of parts in between axes and tangents of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will be

(a) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$ (b) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=2$ (c) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=3$ (d) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=4$

16. The angle of intersection of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and circle $x^2+y^2=ab$, is

(a) $\tan^{-1}\left(\frac{a-b}{ab}\right)$ (b) $\tan^{-1}\left(\frac{a+b}{ab}\right)$ (c) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (d) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

17. Locus of the foot of the perpendicular drawn from the centre upon any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is

(a) $(x^2+y^2)^2=b^2x^2+a^2y^2$ (b) $(x^2+y^2)^2=b^2x^2-a^2y^2$

(c) $(x^2+y^2)^2=a^2x^2-b^2y^2$ (d) $(x^2+y^2)^2=a^2x^2+b^2y^2$

18. If a tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major and minor axes in points *A* and *B* respectively, then the area of ΔOAB is equal to (*O* is centre of the ellipse)

(a) 12 sq. units (b) 48 sq. units (c) 64 sq. units (d) 24 sq. units

19. Tangent is drawn to ellipse $\frac{x^2}{27}+y^2=1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ (where $\theta\in\left(0,\frac{\pi}{2}\right)$). Then the value of θ such that sum of intercepts on axes made by this tangent is minimum, is

(a) $\pi/3$ (b) $\pi/6$ (c) $\pi/8$ (d) $\pi/4$

20. If the tangent at the point $\left(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi\right)$ to the ellipse $16x^2+11y^2=256$ is also a tangent to the circle $x^2+y^2-2x=15$, then the value of ϕ is

(a) $\pm\frac{\pi}{2}$ (b) $\pm\frac{\pi}{4}$ (c) $\pm\frac{\pi}{3}$ (d) $\pm\frac{\pi}{6}$

21. An ellipse passes through the point (4, –1) and its axes are along the axes of co-ordinates. If the line $x+4y-10=0$ is a tangent to it, then its equation is

(a) $\frac{x^2}{100}+\frac{y^2}{5}=1$ (b) $\frac{x^2}{80}+\frac{y^2}{5/4}=1$ (c) $\frac{x^2}{20}+\frac{y^2}{5}=1$ (d) None of these

22. The sum of the squares of the perpendiculars on any tangent to the ellipse $x^2/a^2+y^2/b^2=1$ from two points on the minor axis each distance $\sqrt{a^2-b^2}$ from the centre is

(a) a^2 (b) b^2 (c) $2a^2$ (d) $2b^2$

23. The tangent at a point $P(a\cos\theta, b\sin\theta)$ of an ellipse $x^2/a^2+y^2/b^2=1$, meets its auxiliary circle in two points, the chord joining which subtends a right angle at the centre, then the eccentricity of the ellipse is

(a) $(1+\sin^2\theta)^{-1}$ (b) $(1+\sin^2\theta)^{-1/2}$ (c) $(1+\sin^2\theta)^{-3/2}$ (d) $(1+\sin^2\theta)^{-2}$

24. The locus of the point of intersection of tangents to an ellipse at two points, sum of whose eccentric angles is constant is

(a) A parabola (b) A circle (c) An ellipse (d) A straight line

25. The sum of the squares of the perpendiculars on any tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from two points on the minor axis each at a distance *ae* from the centre is

(a) $2a^2$ (b) $2b^2$ (c) a^2+b^2 (d) a^2-b^2

26. The equation of the circle passing through the points of intersection of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ is

(a) $x^2+y^2=a^2$ (b) $x^2+y^2=b^2$ (c) $x^2+y^2=\frac{a^2b^2}{a^2+b^2}$ (d) $x^2+y^2=\frac{2a^2b^2}{a^2+b^2}$

27. The slope of a common tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and a concentric circle of radius *r* is

(a) $\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$ (b) $\sqrt{\frac{r^2-b^2}{a^2-r^2}}$ (c) $\left(\frac{r^2-b^2}{a^2-r^2}\right)$ (d) $\sqrt{\frac{a^2-r^2}{r^2-b^2}}$

28. The tangents from which of the following points to the ellipse $5x^2+4y^2=20$ are perpendicular

(a) $(1,\ 2\sqrt{2})$ (b) $(2\sqrt{2},\ 1)$ (c) $(2,\sqrt{5})$ (d) $(\sqrt{5},2)$

29. If the normal at the point $P(\theta)$ to the ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$ intersects it again at the point $Q(2\theta)$, then $\cos\theta$ is equal to

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

30. If the normal at any point *P* on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the coordinates axes in *G* and *g* respectively, then PG : *Pg*=

(a) $a:b$ (b) $a^2:b^2$ (c) $b^2:a^2$ (d) $b:a$

31. If α and β are eccentric angles of the ends of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$ is equal to

(a) $\frac{1-e}{1+e}$ (b) $\frac{e-1}{e+1}$ (c) $\frac{e+1}{e-1}$ (d) None of these

32. If the normal at one end of the latus-rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the one end of the minor axis, then

(a) $e^4-e^2+1=0$ (b) $e^2-e+1=0$ (c) $e^2+e+1=0$ (d) $e^4+e^2-1=0$

33. The line $2x+y=3$ cuts the ellipse $4x^2+y^2=5$ at P and Q. If θ be the angle between the normals at these points, then $\tan\theta=$

(a) 1/2 (b) 3/4 (c) 3/5 (d) 5

34. The eccentric angles of extremities of a chord of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are θ_1 and θ_2. If this chord passes through the focus, then

(a) $\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}+\frac{1-e}{1+e}=0$ (b) $\cos\frac{\theta_1-\theta_2}{2}=e.\cos\frac{\theta_1+\theta_2}{2}$

(c) $e=\frac{\sin\theta_1+\sin\theta_2}{\sin(\theta_1+\theta_2)}$ (d) $\cot\frac{\theta_1}{2}.\cot\frac{\theta_2}{2}=\frac{e+1}{e-1}$

35. Let F_1, F_2 be two foci of the ellipse and PT and PN be the tangent and the normal respectively to the ellipse at point *P* then

(a) *PN* bisects $\angle F_1PF_2$ (b) *PT* bisects $\angle F_1PF_2$

(c) *PT* bisects angle $(180^o-\angle F_1PF_2)$ (d) None of these

36. If *CF* is the perpendicular from the centre C of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ on the tangent at any point *P* and *G* is the point when the normal at *P* meets the major axis, then *CF. PG=*

(a) a^2 (b) ab (c) b^2 (d) b^3

37. If $\tan\theta_1\tan\theta_2=-\frac{a^2}{b^2}$, then the chord joining two points θ_1 and θ_2 on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will subtend a right angle at

(a) Focus (b) Centre (c) End of the major axis (d) End of the minor axis

38. If θ and ϕ are the eccentric angles of the ends of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then

(a) $\cos\frac{\theta-\phi}{2}=e\cos\frac{\theta+\phi}{2}$ (b) $\cos\frac{\theta-\phi}{2}+e\cos\frac{\theta+\phi}{2}=0$ (c) $\cos\frac{\theta+\phi}{2}=e\cos\frac{\theta-\phi}{2}$ (d) None of these

39. The locus of the point of intersection of tangents at the ends of semi-conjugate diameter of ellipse is

(a) Parabola (b) Hyperbola (c) Circle (d) Ellipse

40. *AB* is a diameter of $x^2+9y^2=25$. The eccentric angle of *A* is $\pi/6$. Then the eccentric angle of *B* is

(a) $5\pi/6$ (b) $-5\pi/6$ (c) $-2\pi/3$ (d) None of these

41. If the points of intersection of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{p^2}+\frac{y^2}{q^2}=1$ be the extremities of the conjugate diameter of first ellipse, then

(a) $\frac{x^2}{p^2}+\frac{y^2}{q^2}=2$ (b) $\frac{a^2}{p^2}+\frac{b^2}{q^2}=1$ (c) $\frac{a}{p}+\frac{b}{q}=1$ (d) $\frac{a^2}{p^2}+\frac{b^2}{q^2}=2$

42. The equation of the chord of the ellipse $2x^2+5y^2=20$ which is bisected at the point (2, 1)is

(a) $4x+5y+13=0$ (b) $4x+5y=13$ (c) $5x+4y+13=0$ (d) None of these

43. If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are at right angles, then $\frac{x_1x_2}{y_1y_2}$ is equal to

(a) $\frac{a^2}{b^2}$ (b) $-\frac{b^2}{a^2}$ (c) $-\frac{a^4}{b^4}$ (d) $-\frac{b^4}{a^4}$

44. Chords of an ellipse are drawn through the positive end of the minor axis. Then their mid-point lies on

(a) A circle (b) A parabola (c) An ellipse (d) A hyperbola

45. The length of the common chord of the ellipse $\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$ and the circle $(x-1)^2+(y-2)^2=1$ is

(a) Zero (b) One (c) Three (d) Eight

46. The line $y=mx+c$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if c =

(a) $-(2am+bm^2)$ (b) $\frac{(a^2+b^2)m}{\sqrt{a^2+b^2m^2}}$ (c) $-\frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$ (d) $\frac{(a^2-b^2)m}{\sqrt{a^2+b^2}}$

47. The line $lx+my+n=0$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if

(a) $\frac{a^2}{m^2}+\frac{b^2}{l^2}=\frac{(a^2-b^2)}{n^2}$ (b) $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$ (c) $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$ (d) None of these

48. If the line $x\cos\alpha+y\sin\alpha=p$ be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then

(a) $p^2(a^2\cos^2\alpha+b^2\sin^2\alpha)=a^2-b^2$ (b) $p^2(a^2\cos^2\alpha+b^2\sin^2\alpha)=(a^2-b^2)^2$

(c) $p^2(a^2\sec^2\alpha+b^2\operatorname{cosec}^2\alpha)=a^2-b^2$ (d) $p^2(a^2\sec^2\alpha+b^2\operatorname{cosec}^2\alpha)=(a^2-b^2)^2$

49. The equation of the normal at the point (2, 3) on the ellipse $9x^2+16y^2=180$, is

(a) $3y=8x-10$ (b) $3y-8x+7=0$ (c) $8y+3x+7=0$ (d) $3x+2y+7=0$

50. Equations of tangents to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, which cut off equal intercepts on the axes is

(a) $y=x+\sqrt{13}$ (b) $y=-x+\sqrt{13}$ (c) $y=x-\sqrt{13}$ (d) $y=-x-\sqrt{13}$