DIFFERENTIATION ASSIGNMENT

1. If $y = \cot^{-1}(\cos 2x)^{1/2}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ will be

(a) $\left(\frac{2}{3}\right)^{1/2}$ (b) $\left(\frac{1}{3}\right)^{1/2}$ (c) $(3)^{1/2}$ (d) $(6)^{1/2}$

2. The values of *x*, at which the first derivative of the function $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ *w.r.t. x* is $\frac{3}{4}$, are

(a) ± 2 (b) $\pm\frac{1}{2}$ (c) $\pm\frac{\sqrt{3}}{2}$ (d) $\pm\frac{2}{\sqrt{3}}$

3. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval (0, 2), is

(a) 1 (b) 2 (c) 3 (d) 4

4. The set of all those points, where the function $f(x) = \frac{x}{1 + |x|}$ is differentiable, is

(a) $(-\infty, \infty)$ (b) $[0, \infty)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(0, \infty)$

5. Let $f(x + y) = f(x)f(y)$ and $f(x) = 1 + xg(x)G(x)$ where $\lim_{x\to 0} g(x) = a$ and $\lim_{x\to 0} G(x) = b$ then $f'(x)$ is equal to

(a) 1+ *ab* (b) *ab* (c) *a*/*b* (d) None of these

6. *f*(*x*) is a function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$ and *h*(*x*) is a function such that $h(x) = [f(x)]^2 + [g(x)]^2$ and *h*(5) = 11, then the value of *h*(10) is

(a) 0 (b) 1 (c) 10 (d) None of these

7. Let $f(x + y) = f(x)f(y)$ for all *x* and *y*. Suppose that $f(3) = 3$ and $f'(0) = 11$, then $f'(3)$ is given by

(a) 22 (b) 33 (c) 28 (d) None of these

8. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ is

(a) $\frac{x}{\sqrt{1 + x^2}}$ (b) $\frac{-x}{\sqrt{1 + x^2}}$ (c) $\frac{x}{\sqrt{1 - x^2}}$ (d) None of these

9. If $y = (1 + x^{1/4})(1 + x^{1/2})(1 - x^{1/4})$, then $\frac{dy}{dx} =$

(a) 1 (b) – 1 (c) *x* (d) $\sqrt{x}$

10. If $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$, then $f'(a) =$

(a) – 1 (b) 1 (c) 0 (d) *a*

11. $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$, then $\frac{dy}{dx} =$

(a) 1+ *x* (b) $(1 + x)^{-2}$ (c) $-(1 + x)^{-1}$ (d) $-(1 + x)^{-2}$

12. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then $\frac{dy}{dx} =$

(a) $\sqrt{\frac{1 - x^2}{1 - y^2}}$ (b) $\sqrt{\frac{1 - y^2}{1 - x^2}}$ (c) $\sqrt{\frac{x^2 - 1}{1 - y^2}}$ (d) $\sqrt{\frac{y^2 - 1}{1 - x^2}}$

13. Function $y = (x + \sqrt{x^2 + 1})^k$ satisfies

(a) $(x^2 + 1)y' = k^2 y$ (b) $\sqrt{(x^2 + 1)}y' = ky$ (c) $(1 + x^2)y'' + ky' - xy = 0$ (d) $(1 + x^2)y'' + k^2 + xy' = 0$

14. The derivative of $\sqrt{\sqrt{x} + 1}$ is

(a) $\frac{1}{\sqrt{x}(\sqrt{x} + 1)}$ (b) $\frac{-1}{\sqrt{x}\sqrt{x + 1}}$ (c) $\frac{4}{\sqrt{x(\sqrt{x} + 1)}}$ (d) $\frac{1}{4\sqrt{x(\sqrt{x} + 1)}}$

15. If $f(x) = \frac{1}{\sqrt{x^2 + a^2} + \sqrt{x^2 + b^2}}$, then $f'(x)$ is equal to

(a) $\frac{x}{(a^2-b^2)}\left[\frac{1}{\sqrt{x^2+a^2}}-\frac{1}{\sqrt{x^2+b^2}}\right]$ (b) $\frac{x}{(a^2+b^2)}\left[\frac{1}{\sqrt{x^2+a^2}}-\frac{1}{\sqrt{x^2+b^2}}\right]$

(c) $\frac{x}{(a^2-b^2)}\left[\frac{1}{\sqrt{x^2+a^2}}+\frac{1}{\sqrt{x^2+b^2}}\right]$ (d) $(a^2-b^2)\left[\frac{1}{\sqrt{x^2+a^2}}-\frac{2}{\sqrt{x^2+b^2}}\right]$

16. If $\sqrt{(1-x^6)}+\sqrt{(1-y^6)}=a^3(x^3-y^3)$, then $\frac{dy}{dx}=$

(a) $\frac{x^2}{y^2}\sqrt{\frac{1-x^6}{1-y^6}}$ (b) $\frac{y^2}{x^2}\sqrt{\frac{1-y^6}{1-x^6}}$ (c) $\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$ (d) None of these

17. If $y=\sqrt{x+\sqrt{x}}$, then $y\frac{dy}{dx}$ equals

(a) $\frac{2\sqrt{x}+1}{4\sqrt{x}}$ (b) $\frac{\sqrt{x}+1}{2\sqrt{x}}$ (c) $\frac{\sqrt{x}+1}{4x}$ (d) $\frac{x+1}{2\sqrt{x}}$

18. If $y=\sqrt{\sin\sqrt{x}}$, then $\frac{dy}{dx}=$

(a) $\frac{1}{2\sqrt{\cos\sqrt{x}}}$ (b) $\frac{\sqrt{\cos\sqrt{x}}}{2x}$ (c) $\frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{\sin\sqrt{x}}}$ (d) $\frac{1}{2\sqrt{\sin x}}$

19. $\frac{d}{dx}\sqrt{x\sin x}=$

(a) $\frac{\sin x+x\cos x}{2\sqrt{x\sin x}}$ (b) $\frac{\sin x+x\cos x}{\sqrt{x\sin x}}$ (c) $\frac{x\sin x+\cos x}{\sqrt{2\sin x}}$ (d) $\frac{x\sin x+\cos x}{\sqrt{2x\sin x}}$

20. If $y=f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x)=\sin x^2$, then $\frac{dy}{dx}=$

(a) $\frac{6x^2-2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$ (b) $\frac{6x^2-2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$

(c) $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$ (d) $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

21. $\frac{d}{dx}\left[\frac{e^{ax}}{\sin(bx+c)}\right]=$

(a) $\frac{e^{ax}[a\sin(bx+c)+b\cos(bx+c)]}{\sin^2(bx+c)}$ (b) $\frac{e^{ax}[a\sin(bx+c)-b\cos(bx+c)]}{\sin^2(bx+c)}$

(c) $\frac{e^{ax}[a\sin(bx+c)-b\cos(bx+c)]}{\sin^2(bx+c)}$ (d) None of these

22. If $y=b\cos\log\left(\frac{x}{n}\right)^n$, then $\frac{dy}{dx}=$

(a) $-n\ b\sin\log\left(\frac{x}{n}\right)^b$ (b) $n\ b\sin\log\left(\frac{x}{n}\right)^n$ (c) $\frac{-nb}{x}\sin\log\left(\frac{x}{n}\right)^n$ (d) None of these

23. If $y=f\left(\frac{5x+1}{10x^2-3}\right)$ and $f'(x)=\cos x$, then $\frac{dy}{dx}=$

(a) $\cos\left(\frac{5x+1}{10x^2-3}\right)\frac{d}{dx}\left(\frac{5x+1}{10x^2-3}\right)$ (b) $\frac{5x+1}{10x^2-3}\cos\left(\frac{5x+1}{10x^2-3}\right)$

(c) $\cos\left(\frac{5x+1}{10x^2-3}\right)$ (d) None of these

24. $\frac{d}{dx}\left(x^3\tan^2\frac{x}{2}\right)=$

(a) $x^3\tan\frac{x}{2}.\sec^2\frac{x}{2}+3x\tan^2\frac{x}{2}$ (b) $x^3\tan\frac{x}{2}.\sec^2\frac{x}{2}+3x^2\tan^2\frac{x}{2}$

(c) $x^2 \tan^2 \frac{x}{2}.\sec^2 \frac{x}{2} + 3x^2 \tan^2 \frac{x}{2}$ (d) None of these

25. $\frac{d}{dx}(\tan a^{1/x}) =$

(a) $\sec^2(a^{1/x}) \cdot \frac{(a^{1/x}.\log a)}{x^2}$ (b) $\sec^2(a^{1/x}).(a^{1/x}.\log a)$ (c) $\frac{\sec x.\log a}{x^2}$ (d) $-\frac{\sec^2(a^{1/x}).(a^{1/x}.\log_e a)}{x^2}$

26. If $x^2 + y^2 = t - \frac{1}{t}$, $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $\frac{dy}{dx}$ equals

(a) $1/xy^3$ (b) $1/x^3y$ (c) $-1/x^3y$ (d) $-1/xy^3$

27. If $f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx} =$

(a) $\frac{9\cos(\log x)}{x(3-2x)^2}$ (b) $\frac{9\cos\left(\log\frac{2x+3}{3-2x}\right)}{x(3-2x)^2}$ (c) $\frac{9\sin\left(\log\frac{2x+3}{3-2x^2}\right)}{(3-2x)^2}$ (d) None of these

28. $\frac{dy}{dx}$ of $\log(xy) = x^2 + y^2$ is

(a) $\frac{y(2x^2-1)}{x(1-2y^2)}$ (b) $\frac{y(2x^2+1)}{x(1+2y^2)}$ (c) $\frac{x(2x^2-1)}{y(2y^2-1)}$ (d) $\frac{y(2x^2-1)}{x(2y^2-1)}$

29. $(x-y)e^{x/(x-y)} = k$, then

(a) $(y-2x)\frac{dy}{dx} + 3x - 2y = 0$ (b) $y\frac{dy}{dx} + x - 2y = 0$ (c) $a\left(y\frac{dy}{dx} + x - 2y\right) = 1$ (d) None of these

30. If $y = (x^x)^x$, then $\frac{dy}{dx} =$

(a) $(x^x)^x(1 + 2\log x)$ (b) $(x^x)^x(1 + \log x)$ (c) $x(x^x)^x(1 + 2\log x)$ (d) $x(x^x)^x(1 + \log x)$

31. If $y = (x\log x)^{\log\log x}$, then $\frac{dy}{dx} =$

(a) $(x\log x)^{\log\log x}\left\{\frac{1}{x\log x}(\log x + \log\log x) + (\log\log x)\left(\frac{1}{x} + \frac{1}{x\log x}\right)\right\}$

(b) $(x\log x)^{x\log x}\log\log x\left[\frac{2}{\log x} + \frac{1}{x}\right]$

(c) $(x\log x)^{x\log x}\frac{\log\log x}{x}\left[\frac{1}{\log x} + 1\right]$

(d) None of these

32. If $y = \left(1 + \frac{1}{x}\right)^x$, then $\frac{dy}{dx} =$

(a) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right) - \frac{1}{1+x}\right]$ (b) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)\right]$

(c) $\left(x+\frac{1}{x}\right)^x\left[\log(x-1) - \frac{x}{1+x}\right]$ (d) $\left(x+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right) + \frac{1}{1+x}\right]$

33. If $y = x^{(x^x)}$, then $\frac{dy}{dx} =$

(a) $y[x^x(\log ex).\log x + x^x]$ (b) $y[x^x(\log ex).\log x + x]$ (c) $y[x^x(\log ex).\log x + x^{x-1}]$ (d) $y[x^x(\log_e x).\log x + x^{x-1}]$

34. If $y = \frac{a^{\cos^{-1}x}}{1 + a^{\cos^{-1}x}}$ and $z = a^{\cos^{-1}x}$, then $\frac{dy}{dz} =$

(a) $\frac{1}{1+a^{\cos^{-1}x}}$ (b) $-\frac{1}{1+a^{\cos^{-1}x}}$ (c) $\frac{1}{(1+a^{\cos^{-1}x})^2}$ (d) None of these

35. Let the function $y = f(x)$ be given by $x = t^5 - 5t^3 - 20t + 7$ and $y = 4t^3 - 3t^2 - 18t + 3$, where $t \in (-2, 2)$. Then $f'(x)$ at $t = 1$ is

(a) $\frac{5}{2}$ (b) $\frac{2}{5}$ (c) $\frac{7}{5}$ (d) None of these

36. If $y=\sqrt{x}^{\sqrt{x}^{\sqrt{x}.....\infty}}$, then $\frac{dy}{dx}=$

(a) $\frac{y^2}{2x-2y\log x}$ (b) $\frac{y^2}{2x+\log x}$ (c) $\frac{y^2}{2x+2y\log x}$ (d) None of these

37. If $y=x+\cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x+.....}}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{y}{2y-x}$ (b) $\frac{y}{2y+x}$ (c) $\frac{y}{y-2x}$ (d) $\frac{y}{y+2x}$

38. If $y=\cfrac{x}{a+\cfrac{x}{b+\cfrac{x}{a+\cfrac{x}{b+.....}}}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{b}{a(b+2y)}$ (b) $\frac{b}{b+2y}$ (c) $\frac{a}{b(b+2y)}$ (d) None of these

39. If $y=\cfrac{\sin x}{1+\cfrac{\cos x}{1+\cfrac{\sin x}{1+\cos x.....\infty}}}$, then $\frac{dy}{dx}=$

(a) $\frac{(1+y)\cos x+\sin x}{1+2y+\cos x-\sin x}$ (b) $\frac{(1+y)\cos x-\sin x}{1+2y+\cos x+\sin x}$ (c) $\frac{(1+y)\cos x+\sin x}{1+2y+\cos x+\sin x}$ (d) None of these

40. If $f(x)=\frac{1}{1-x}$, then the derivative of the composite function $f[f\{f(x)\}]$ is equal to

(a) 0 (b) 1/2 (c) 1 (d) 2

41. If $u=f(x^3), v=g(x^2), f'(x)=\cos x$ and $g'(x)=\sin x$ then $\frac{du}{dv}$ is

(a) $\frac{3}{2}x.\cos x^3.\text{cosec } x^2$ (b) $\frac{2}{3}\sin x^3.\sec x^2$ (c) $\tan x$ (d) None of these

42. Let $f(x)=e^x, g(x)=\sin^{-1}x$ and $h(x)=f(g(x))$, then $h'(x)/h(x)=$

(a) $e^{\sin^{-1}x}$ (b) $1/\sqrt{1-x^2}$ (c) $\sin^{-1}x$ (d) $1/(1-x^2)$

43. Differential coefficient of $\frac{\tan^{-1}x}{1+\tan^{-1}x}$ *w.r.t.* $\tan^{-1}x$ is

(a) $\frac{1}{1+\tan^{-1}x}$ (b) $\frac{-1}{1+\tan^{-1}x}$ (c) $\frac{1}{(1+\tan^{-1}x)^2}$ (d) $\frac{-1}{2(1+\tan^{-1}x)^2}$

44. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$, is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

45. Differentiation of $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}(2x\sqrt{1-x^2})$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) – 1

46. Differentiation of $\sin^{-1}(2ax\sqrt{1-a^2x^2})$ with respect to $\sqrt{1-a^2x^2}$ is

(a) 2 (b) ax (c) $\frac{2}{ax}$ (d) $-\frac{2}{ax}$

47. Differentiation of $\tan^{-1}\left(\frac{1+ax}{1-ax}\right)$ with respect to $\sqrt{1+a^2x^2}$ is

(a) $\frac{1}{ax\sqrt{1+ax}}$ (b) $\frac{1}{\sqrt{1+ax}}$ (c) $\frac{1}{ax\sqrt{1+a^2x^2}}$ (d) $\frac{1}{ax\sqrt{1-a^2x^2}}$

48. The value of derivative of $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ *w.r.t.* to $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ at $x=\frac{1}{2}$ equals

(a) 1 (b) – 1 (c) 0 (d) None of these

49. If $x=f_1(t)$ and $y=f_2(t)$, then $\frac{d^2y}{dx^2}=$

(a) $\frac{f_1'f_2''-f_2'f_1''}{(f_1')^2}$ (b) $\frac{f_1'f_2''-f_2'f_1''}{(f_1')^3}$ (c) $\frac{f_1''(t)}{f_2''(t)}$ (d) $\frac{-f_1''(t)}{f_2''(t)}$

50. If $y^2=p(x)$ is a polynomial of degree three, then $2\frac{d}{dx}\left\{y^3.\frac{d^2y}{dx^2}\right\}=$

(a) $p'''(x)+p'(x)$ (b) $p''(x).p'''(x)$ (c) $p(x).p'''(x)$ (d) Constant