

DEFINITE INTEGRAL

1. If $(n - m)$ is odd and $|m| \neq |n|$, then $\int_0^\pi \cos mx \sin nx \, dx$ is
 (a) $\frac{2n}{n^2 - m^2}$ (b) 0 (c) $\frac{2n}{m^2 - n^2}$ (d) $\frac{2m}{n^2 - m^2}$
2. The value of the definite integral $\int_0^1 \frac{dx}{x^2 + 2x \cos \alpha + 1}$ for $0 < \alpha < \pi$ is equal to
 (a) $\sin \alpha$ (b) $\tan^{-1}(\sin \alpha)$ (c) $\alpha \sin \alpha$ (d) $\frac{\alpha}{2}(\sin \alpha)^{-1}$
3. If $f(y) = e^y, g(y) = y; y > 0$ and $F(t) = \int_0^t f(t-y)g(y)dy$, then
 (a) $F(t) = 1 - e^{-t}(1+t)$ (b) $F(t) = e^t - (1+t)$ (c) $F(t) = te^t$ (d) $F(t) = te^{-t}$
4. If $I_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx$, then
 (a) $I_n = \frac{n\pi}{2}$ (b) $I_1, I_2, I_3, I_4, \dots, I_n, \dots$ are in A.P.
 (c) $\sin(I_{16}) = 0$ (d) All of these
5. If $n \in \mathbb{N}$ and $\int_0^1 e^x(x-1)^n dx = 2e - 5$, then $n =$
 (a) 1 (b) 2 (c) 3 (d) None of these
6. The value of $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x) d(x - [x])$, (where $[.]$ denotes the greatest integer function) is
 (a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $\frac{2}{n-1}$ (d) None of these
7. The absolute value of $\int_{10}^{19} \frac{\cos x}{1+x^8} dx$ is
 (a) Less than 10^{-7} (b) More than 10^{-7} (c) Less than 10^{-6} (d) Both (a) and (c)
8. $\int_0^{\pi/4} \sin x(x - [x]) dx$ is equal to
 (a) $\frac{1}{2}$ (b) $1 - \frac{1}{\sqrt{2}}$ (c) 1 (d) None of these
9. $\int_{-1}^{10} \text{sgn}(x - [x]) dx$ equals
 (a) 10 (b) 11 (c) 9 (d) $\frac{11}{2}$
10. If $I_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$ and $a_n = \int_0^{\pi/2} \left(\frac{\sin n\theta}{\sin \theta}\right)^2 d\theta$, then $a_{n+1} - a_n =$
 (a) I_n (b) $2I_n$ (c) I_{n+1} (d) 0
11. If $f'(x) = f(x) + \int_0^1 f(x) dx$ and given $f(0) = 1$ then $f(x) =$
 (a) $\frac{e^x}{2-e} + \left(\frac{1+e}{1-e}\right)$ (b) $\frac{2e^x}{3-e} + \left(\frac{1-e}{1+e}\right)$ (c) $\frac{e^x}{2-e}$ (d) $\frac{2e^x}{3-e}$
12. On the interval $\left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$, the greatest value of the function $f(x) = \int_{5\pi/3}^x (6 \cos t - 2 \sin t) dt =$
 (a) $3\sqrt{3} + 2\sqrt{2} + 1$ (b) $3\sqrt{3} - 2\sqrt{2} - 1$ (c) Does not exist (d) None of these
13. If $u_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx$, then $u_2 - u_1, u_3 - u_2, u_4 - u_3, \dots$ are in
 (a) A.P. (b) G.P. (c) H.P. (d) None
14. If $f(x) = \begin{cases} x, & \text{for } x < 1 \\ x-1, & \text{for } x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x) dx$ is equal to

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(a) 1

(b) $\frac{4}{3}$

(c) $\frac{5}{3}$

(d) $\frac{5}{2}$

15. If $I = \int_0^{1/2} \frac{1}{\sqrt{1-x^{2n}}} dx$, then

(a) $I \leq \frac{\pi}{6}$

(b) $I \geq \frac{\pi}{2}$

(c) $I \geq 0$

(d) All of these

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