Complex number Assignment

1. If z_1 and z_2 are two complex numbers satisfying the equation $\left|\frac{z_1 + z_2}{z_1 - z_2}\right| = 1$, then $\frac{z_1}{z_2}$ is a number which is

(a) Positive real (b) Negative real (c) Zero or purely imaginary (d) None of these

2. If $z(1+a) = b + ic$ and $a^2 + b^2 + c^2 = 1$, then $\frac{1+iz}{1-iz} =$

(a) $\frac{a+ib}{1+c}$ (b) $\frac{b-ic}{1+a}$ (c) $\frac{a+ic}{1+b}$ (d) None of these

3. Given that the equation $z^2 + (p+iq)z + r + is = 0$, where, p, q, r, s are real and non-zero has a real root, then

(a) $pqr = r^2 + p^2 s$ (b) $prs = q^2 + r^2 p$ (c) $qrs = p^2 + s^2 q$ (d) $pqs = s^2 + q^2 r$

4. If $\sum_{k=0}^{100} i^k = x + iy$, then the value of x and y are

(a) $x = -1, \ y = 0$ (b) $x = 1, \ y = 1$ (c) $x = 1, \ y = 0$ (d) $x = 0, \ y = 1$

5. Let $\frac{1-ix}{1+ix} = a - ib$ and $a^2 + b^2 = 1$, where a and b are real, then $x =$

(a) $\frac{2a}{(1+a)^2 + b^2}$ (b) $\frac{2b}{(1+a)^2 + b^2}$ (c) $\frac{2a}{(1+b)^2 + a^2}$ (d) $\frac{2b}{(1+b)^2 + a^2}$

6. If $\frac{(p+i)^2}{2p-i} = \mu + i\lambda$, then $\mu^2 + \lambda^2$ is equal to

(a) $\frac{(p^2+1)^2}{4p^2-1}$ (b) $\frac{(p^2-1)^2}{4p^2-1}$ (c) $\frac{(p^2-1)^2}{4p^2+1}$ (d) $\frac{(p^2+1)^2}{4p^2+1}$

7. If $(1+i)(1+2i)(1+3i)......(1+ni) = a + ib$, then 2 . 5 . 10.........$(1+n^2)$ is equal to

(a) $a^2 - b^2$ (b) $a^2 + b^2$ (c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

8. Given $z = \frac{q+ir}{1+p}$, then $\frac{p+iq}{1+r} = \frac{1+iz}{1-iz}$ if

(a) $p^2 + q^2 + r^2 = 1$ (b) $p^2 + q^2 + r^2 = 2$ (c) $p^2 + q^2 - r^2 = 1$ (d) None of these

9. The equation $z^2 = \bar{z}$ has

(a) No solution (b) Two solutions

(c) Four solutions (d) An infinite number of solutions

10. If $z_1 = 9y^2 - 4 - 10ix, z_2 = 8y^2 - 20i$, where $z_1 = \bar{z}_2$, then $z = x + iy$ is equal to

(a) $-2 + 2i$ (b) $-2 \pm 2i$ (c) $-2 \pm i$ (d) None of these

11. If α is a complex constant such that $\alpha z^2 + z + \bar{\alpha} = 0$ has a real root then

(a) $\alpha + \bar{\alpha} = 1$ (b) $\alpha + \bar{\alpha} = 0$

(c) $\alpha + \bar{\alpha} = -1$ (d) The absolute value of the real root is 1

12. If z is a complex number, then the minimum value of $|z| + |z - 1|$ is

(a) 1 (b) 0 (c) 1/2 (d) None of these

13. The maximum value of $|z|$ where z satisfies the condition $\left|z + \frac{2}{z}\right| = 2$ is

(a) $\sqrt{3} - 1$ (b) $\sqrt{3} + 1$ (c) $\sqrt{3}$ (d) $\sqrt{2} + \sqrt{3}$

14. If $|z + 4| \le 3$, then the greatest and the least value of $|z + 1|$ are

(a) 6, – 6 (b) 6, 0 (c) 7, 2 (d) 0, – 1

15. Let z be a complex number, then the equation $z^4 + z + 2 = 0$ cannot have a root, such that

(a) $|z| < 1$ (b) $|z| = 1$ (c) $|z| > 1$ (d) None of these

16. Let z and w be two complex numbers such that $|z| \le 1$, $|w| \le 1$ and $|z + iw| = |z - i\bar{w}| = 2$. Then z is equal to

(a) 1 or i (b) i or $-i$ (c) 1 or – 1 (d) i or – 1

17. If | z_1 | = | z_2 | = ... = | z_n | = 1, then the value of | $z_1 + z_2 + z_3 +$ .... $+ z_n$ |=

(a) 1 (b) | z_1 | + | z_2 | + ... + | z_n | (c) $\left|\frac{1}{z_1}+\frac{1}{z_2}+.....+\frac{1}{z_n}\right|$ (d) None of these

18. If z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{(z_1+z_2)}{(z_1-z_2)}$ may be

(a) Purely imaginary (b) Real and positive (c) Real and negative (d) None of these

19. For any two complex numbers z_1 and z_2 and any real numbers *a* and *b*; $|(az_1 - bz_2)|^2 + |(bz_1 + az_2)|^2 =$

(a) $(a^2+b^2)(|z_1|+|z_2|)$ (b) $(a^2+b^2)(|z_1|^2+|z_2|^2)$ (c) $(a^2+b^2)(|z_1|^2-|z_2|^2)$ (d) None of these

20. If $|a_k| < 1, \lambda_k \geq 0$ for *k* = 1, 2, ... *n* and $\lambda_1 + \lambda_2 + ... + \lambda_n = 1$, then the value of $|\lambda_1 a_1 + \lambda_2 a_2 + ... + \lambda_n a_n|$ is

(a) Equal to one (b) Greater than one (c) Zero (d) Less than one

21. If z_1, z_2, z_3, z_4 are roots of the equation $a_0 z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4 = 0$, where a_0, a_1, a_2, a_3 and a_4 are real, then

(a) $\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4$ are also roots of the equation (b) z_1 is equal to at least one of $\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4$

(c) $-\bar{z}_1, -\bar{z}_2, -\bar{z}_3, -\bar{z}_4$ are also roots of the equation (d) None of these

22. If *z* satisfies | *z* + 1 | < | *z* – 2 |, then *w* = 3*z* + 2 + *i*

(a) | *w* + 1 | < | *w* – 8 | (b) | *w* + 1 | < | *w* – 7 | (c) $w + \bar{w} > 7$ (d) | *w* + 5 | < | *w* – 4 |

23. | z_1 – 1 | < 1, | z_2 – 2 | < 2, | z_3 – 3 | < 3 then | $z_1 + z_2 + z_3$ |

(a) Is less than 6 (b) Is more than 3 (c) Is less than 12 (d) Lies between 6 and 12

24. If $|z - 4 + 3i| \leq 1$ and *m* and *n* be the least and greatest values of | *z* | and *K* be the least value of $\frac{x^4 + x^2 + 4}{x}$ on the interval $(0, \infty)$, then *K* =

(a) *n* (b) *m* (c) *m* + *n* (d) None of these

25. The system of equations $|z + 1 - i| = \sqrt{2}$ and | *z* | = 3 has

(a) No solution (b) One solution (c) Two solutions (d) None of these

26. If complex number $z = x + iy$ is taken such that the amplitude of fraction $\frac{z-1}{z+1}$ is always $\frac{\pi}{4}$, then

(a) $x^2 + y^2 + 2y = 1$ (b) $x^2 + y^2 - 2y = 0$ (c) $x^2 + y^2 + 2y = -1$ (d) $x^2 + y^2 - 2y = 1$

27. If $z_1 = 10 + 6i$, $z_2 = 4 + 6i$ and *z* is a complex number such that *amp* $\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{4}$, then the value of | *z* – 7 – 9*i* | is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

28. If z_1 = 8 + 4 *i*, z_2 = 6 + 4 *i* and *arg* $\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{4}$, then *z* satisfies

(a) | *z* – 7– 4*i* | = 1 (b) | *z* – 7– 5*i* | = $\sqrt{2}$ (c) | *z* – 4*i* | = 8 (d) | *z* – 7*i* | = $\sqrt{18}$

29. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then *arg* $\left(\frac{z_1}{z_4}\right) + arg\left(\frac{z_2}{z_3}\right)$ equals

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

30. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $R(z_1 \bar{z}_2) = 0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies

(a) | w_1 | = 1 (b) | w_2 | = 1 (c) $R(w_1 \bar{w}_2) = 0$ (d) All the above

31. If z_1, z_2, z_3 be three non-zero complex numbers, such that $z_2 \neq z_1$, $a = |z_1|$, $b = |z_2|$ and $c = |z_3|$.

Suppose that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then $arg\left(\frac{z_3}{z_2}\right)$ is equal to

(a) $arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right)^2$ (b) $arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right)$ (c) $arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^2$ (d) $arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)$

32. If *amp* $\frac{z-2}{2z+3i} = 0$ and $z_0 = 3+4i$ then

(a) $z_0\bar{z} + \bar{z}_0 z = 12$ (b) $z_0 z + \bar{z}_0\bar{z} = 12$ (c) $z_0\bar{z} + \bar{z}_0 z = 0$ (d) None of these

33. The principal value of the *arg* (*z*) and | *z* | of the complex number $z = 1+\cos\left(\frac{11\pi}{9}\right) + i\sin\frac{11\pi}{9}$ are respectively

(a) $\frac{11\pi}{8}, 2\cos\left(\frac{\pi}{18}\right)$ (b) $-\frac{7\pi}{18}, -2\cos\left(\frac{11\pi}{18}\right)$ (c) $\frac{2\pi}{9}, 2\cos\left(\frac{7\pi}{18}\right)$ (d) $-\frac{\pi}{9}, -2\cos\left(\frac{\pi}{18}\right)$

34. If *amp* $(z_1 z_2) = 0$ and $|z_1|=|z_2|=1$ then

(a) $z_1 + z_2 = 0$ (b) $z_1 z_2 = 1$ (c) $z_1 = \bar{z}_2$ (d) None of these

35. If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ then

(a) $\frac{z_1}{z_2}$ is purely real (b) $\frac{z_1}{z_2}$ is purely imaginary (c) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$ (d) $amp\ \frac{z_1}{z_2} = \frac{\pi}{2}$

36. Let $z_1 = \frac{(\sqrt{3}+i)^2.(1-\sqrt{3}\,i)}{1+i}, z_2 = \frac{(1+\sqrt{3}i)^2.(\sqrt{3}-i)}{1-i}$. Then

(a) $|z_1| = |z_2|$ (b) *amp* z_1 + *amp* $z_2 = 0$ (c) $3|z_1| = |z_2|$ (d) 3 *amp* z_1 + *amp* $z_2 = 0$

37. If z_1 and z_2 both satify $z + \bar{z} = 2|z-1|$ and *arg* $(z_1 - z_2) = \frac{\pi}{4}$, then the imaginary part of (z_1+z_2) is

(a) 0 (b) 1 (c) 2 (d) None of these

38. If $z = \frac{(z_1 + \bar{z}_2)z_1}{z_2\bar{z}_1}$, where $z_1 = 1+2i$ and $z_2 = 1-i$, then

(a) $|z| = \frac{1}{2}\sqrt{26}$, $arg\ z = -\pi + \tan^{-1}\frac{19}{17}$ (b) $|z| = \frac{1}{2}\sqrt{26}$, $arg\ z = \tan^{-1}\frac{19}{17}$

(c) $|z| = \frac{1}{2}\sqrt{15}$, $arg\ z = \tan^{-1}\frac{19}{17}$ (d) $arg\ z = -\pi + \tan^{-1}\frac{19}{17}$; $|z| = \frac{1}{3}\sqrt{26}$

39. If $(a_1 + ib_1)(a_2 + ib_2)....(a_n + ib_n) = A + iB$, then $\sum_{i=1}^{n} \tan^{-1}\left(\frac{b_i}{a_i}\right)$ is equal to

(a) $\frac{B}{A}$ (b) $\tan\left(\frac{B}{A}\right)$ (c) $\tan^{-1}\left(\frac{B}{A}\right)$ (d) $\tan^{-1}\left(\frac{A}{B}\right)$

40. Let *z* be a complex number satisfying $|z - 5i| \le 1$ such that *amp z* is minimum. Then *z* is equal to

(a) $\frac{2\sqrt{6}}{5} + \frac{24i}{5}$ (b) $\frac{24}{5} + \frac{2\sqrt{6}i}{5}$ (c) $\frac{2\sqrt{6}}{5} - \frac{24i}{5}$ (d) None of these

41. If ω is a complex number satisfying $\left|\omega + \frac{1}{\omega}\right| = 2$, then maximum distance of ω from origin is

(a) $2+\sqrt{3}$ (b) $1+\sqrt{2}$ (c) $1+\sqrt{3}$ (d) None of these

42. If $|z - 25i| \le 15$, then | max. *amp* (*z*) – min. *amp* (*z*) | =

(a) $\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\pi - 2\cos^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{\pi}{2} + \cos^{-1}\left(\frac{3}{5}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{3}{5}\right)$

43. If z_1, z_2 are two complex numbers such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$ and $iz_1 = kz_2$, where $k \in R$, then the angle between $z_1 - z_2$ and $z_1 + z_2$ is

(a) $\tan^{-1}\left(\frac{2k}{k^2+1}\right)$ (b) $\tan^{-1}\left(\frac{2k}{1-k^2}\right)$ (c) $-2\tan^{-1}k$ (d) $2\tan^{-1}k$

44. If at least one value of the complex number $z = x + iy$ satisfy the condition $|z + \sqrt{2}| = a^2 - 3a + 2$ and the inequality $|z + i\sqrt{2}| < a^2$, then

(a) $a > 2$ (b) $a = 2$ (c) $a < 2$ (d) None of these

45. The maximum distance from the origin of coordinates to the point *z* satisfying the equation $\left|z + \frac{1}{z}\right| = a$ is

(a) $\frac{1}{2}(\sqrt{a^2+1} + a)$ (b) $\frac{1}{2}(\sqrt{a^2+2} + a)$ (c) $\frac{1}{2}(\sqrt{a^2+4} + a)$ (d) None of these

46. Let *a* be a complex number such that $|a| < 1$ and z_1, z_2 .... be vertices of a polygon such that $z_k = 1 + a + a^2 + ... + a^{k-1}$. Then the vertices of the polygon lie within a circle

(a) $|z - a| = a$ (b) $\left|z - \frac{1}{1-a}\right| = |1-a|$ (c) $\left|z - \frac{1}{1-a}\right| = \frac{1}{|1-a|}$ (d) $|z - (1-a)| = |1-a|$

47. If *a, b, c* and *u, v, w* are complex numbers representing the vertices of two traingles such that $c = (1-r)a + rb$ and $w = (1-r)u + rv$, where *r* is *a* complex number, then the two triangles

(a) Have the same area (b) Are similar (c) Are congruent (d) None of these

48. If z_1, z_2, z_3, z_4 are the affixes of four points in the Argand plane and *z* is the affix of a point such that $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$, then z_1, z_2, z_3, z_4 are

(a) Concyclic (b) Vertices of a parallelogram (c) Vertices of a rhombus (d) In a straight line

49. *ABCD* is a rhombus. Its diagonals *AC* and *BD* intersect at the point *M* and satisfy *BD* =2*AC*. If the points *D* and *M* represents the complex numbers 1 + *i* and 2 – *i* respectively, then *A* represents the complex number

(a) $3 - \frac{1}{2}i$ or $1 - \frac{3}{2}i$ (b) $\frac{3}{2} - i$ or $\frac{1}{2} - 3i$ (c) $\frac{1}{2} - i$ or $1 - \frac{1}{2}i$ (d) None of these

50. Suppose Z_1, Z_2, Z_3 are the vertices of an equilateral triangle inscribed in the circle $|Z| = 2$. If $Z_1 = 1 + i\sqrt{3}$, then values of Z_3 and Z_2 are respectively

(a) $-2, 1 - i\sqrt{3}$ (b) $2, 1 + i\sqrt{3}$ (c) $1 + i\sqrt{3}, -2$ (d) None of these